Calculation of the Viscous Drag on Submerged Bodies From the Far Field Behavior of the Flow

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LONG-TERM GOALS

To develop formulas to evaluate the hydrodynamic forces acting on submerged solid bodies using only the far field behavior of the flow past the body. These formulas will eliminate the need to resolve the local behavior of the flow in the vicinity of the bodies. Once these formulas are developed, the interactions between submerged solid bodies both with and without a free surface may be analyzed without the need to resolve the local fluid velocity.

OBJECTIVES

Lighthill [L1] [L2] used a decomposition of the fluid velocity based on a Newtonian potential to relate the Morison equation to the wave induced loads on submerged bodies. In order to obtain a more direct relation between the hydrodynamic forces and the state of the fluid, different representations of the fluid velocity are required. Accordingly, our objectives are:

- To improve the Lighthill approach by considering potentials that are more appropriate for studying fluid flows, such as the Stokes and the Oseen fundamental tensors.
- To obtain exact or asymptotic formulas that relate the far field behavior of the fluid flow to the stress vector.
- To use these relations for the evaluation of the hydrodynamic forces on submerged solid bodies.

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APPROACH

The basic principles of the mechanics of fluids and solids used by Lighthill, [L1] [L2] were:
a) Decomposing the displacement of the fluid into a pure straining component and a pure rotational component; and b) Obtaining a Helmholtz decomposition of the velocity field into an irrotational and a solenoidal component. The Lighthill decomposition may be used to express symbolically the loads on submerged bodies; but the numerical computation of these loads is not possible due to absence of boundary conditions for this decomposition. One of the limitations is the Newtonian potential. This problem may be avoided in the linear case by using the Stokes or Oseen fundamental solution and by focusing instead on the stress vector.

The approach will develop formulas for the direct evaluation of the viscous shear stresses for steady nonlinear flows. A solid body moving with a constant velocity ***-U*** in a fluid at rest at infinity is analyzed. The Navier Stokes equations for the steady flow of an incompressible fluid of unit density are the governing equations of motion for the fluid occupying the region exterior to the solid body are given by [B]:

$$\left(\boldsymbol{U} + \boldsymbol{v}\right) \cdot \nabla\boldsymbol{v} = -\mu\nabla \times \boldsymbol{\omega} - \nabla p - g\boldsymbol{k} \;; \quad \nabla \cdot \boldsymbol{v} = 0$$

where μ is the dynamic fluid viscosity, **ω** is the vorticity, and *p* is the dynamic pressure. On the surface of the solid body B, the no slip boundary condition is assumed to hold $\boldsymbol{v}(\mathbf{x}) = -\boldsymbol{U}$ for $\mathbf{x} \in$ B.

The approach uses potential theory identities to relate the far field behavior of the dynamic pressure to the total forces acting on the submerged solid. This is a departure from Lighthill's approach that considers the velocity field.

WORK COMPLETED

On a solid body with angular velocity , the stress vector may be written as

$$T\boldsymbol{n} = \mu(\boldsymbol{\omega} - 2 \quad)\times\boldsymbol{n} - p\boldsymbol{n} \tag{1}$$

where ***n*** is the inward directed unit vector normal to the surface of the solid body B. The total surface forces acting on the submerged solid body are given by

$$F = \int_B T\boldsymbol{n}\, dS$$

For the Stokes and Oseen problem, Schuster [S] numerically evaluated the stress vector on the boundary of the solid, given the far field velocity by solving a Fredholm equation of the first kind. The kernel of the integral equation is obtained from the fundamental solution tensor of the corresponding equation. Relations between the velocity at infinity and the stress vector appear to be limited to only the linear problems. Consequently, the dynamic pressure will be considered.

For the nonlinear steady flow problem, the following new formula for dynamic pressure was derived:

$$
\begin{aligned}
4\ \ p(\mathbf{x}) \; = \; & \int_B T(\mathbf{y})\boldsymbol{n}(\mathbf{y})\cdot\nabla\frac{1}{|\mathbf{x}-\mathbf{y}|}\,dS(\mathbf{y}) \; - \; \int (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\cdot\nabla\frac{1}{|\mathbf{x}-\mathbf{y}|}\,d\mathbf{y} \\
& - \int_B (\mathbf{U}\cdot\nabla)\boldsymbol{v}\cdot\boldsymbol{n}(\mathbf{y})\frac{1}{|\mathbf{x}-\mathbf{y}|}\,dS(\mathbf{y}) \; - \; \int_B g\frac{\boldsymbol{k}\cdot\boldsymbol{n}}{|\mathbf{x}-\mathbf{y}|}\,dS(\mathbf{y})
\end{aligned}
\tag{2}
$$

The relation between the far field behavior of the dynamic pressure and the stress vector may be obtained as follows. Multiplying (2) for p by $\mathbf{x}/R$ and integrating over the sphere of radius R, the last two integrals vanish. In addition

$$
\int_{\mathbf{x}\ \ = R} \left(\frac{\mathbf{x}}{R} \otimes \nabla \frac{1}{\mathbf{x}-\mathbf{y}} \right) dS(\mathbf{x}) \; = \; \frac{1}{3}4\ \ I + O\left(\frac{1}{R}\right)
$$

where I is the 3x3 identity matrix. The volume integral in (2) for the pressure formula decays according to the results given by Finn [F]. The hydrodynamic forces on the submerged solid body may then be computed from

$$
\int_B T\boldsymbol{n}\,dS \; = \; 3\ \lim_{R} \int_{\mathbf{x}\ \ = R} p(\mathbf{x})\frac{\mathbf{x}}{R}\,dS(\mathbf{x})
\tag{3}
$$

The details of this derivation are given in [GHT].

RESULTS

The numerical calculations by Schuster [S] for the linear case show that it is possible to relate the solid body velocity to the stress vector. The relations that one obtains require the fundamental solution of the Stokes or Oseen problem, and may not be accomplished using a Newtonian potential as suggested by Lighthill [L1] [L2]. The numerical solution of the integral equations that arise requires the characterization of the null space that is given by Fisher, Hsiao and Wendland [F], [HW],[FHW].

In the nonlinear regime, it does not appear possible to relate the solid body velocity to the stress vector. In part, this is due to the coupling of the kinematic boundary conditions for the scalar and vector potentials used in the Helmholtz decomposition of velocity vector fields. Consequently, relations between the stress vector and thermodynamic variables (*e.g.*, dynamic pressure) are required. An example of this relation for steady flow is given by (3).

IMPACT/APPLICATIONS

The new formulas (2) and (3) appear to provide a method for computing fluid drag forces that is not possible using the Lighthill velocity decomposition.

The significance of (1) is that the stress vector may be related pointwise to the vorticity of the fluid. Consequently, (1) may be used to derive rigorous or semi-rigorous models for the study of sediment transport.

TRANSITIONS

These results lead to the development of efficient numerical methods for solving Fredholm integral equations of the first kind. Equation (3) offers an appealing alternative to far field velocity [LI1] or vorticity [R] methods for computing the hydrodynamic force on submerged solid bodies that could have substantial impact on the analysis of loading on structures.

RELATED PROJECTS

The results of this research could be adapted and reformulated to analyze sediment transport. These results could also be used to determine the boundary data needed in numerical algorithms based on vortex elements for computational fluid mechanics.

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